Black hole mergers: beyond general relativity

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Why test GR?

$$G_{ab} = 8\pi \hat{T}_{ab}$$

General relativity successful but incomplete

- Can't have mix of quantum/classical
- GR not renormalizable
- GR+QM=new physics (e.g. BH information paradox)

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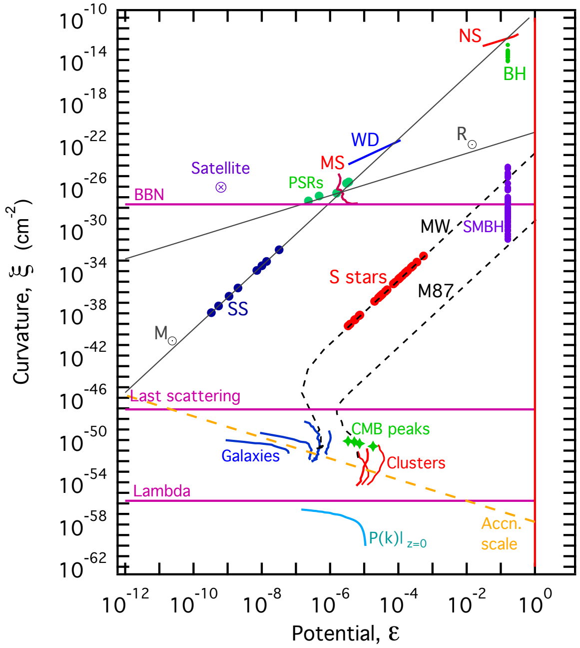
Empiricism

Ultimate test of theory: ask nature

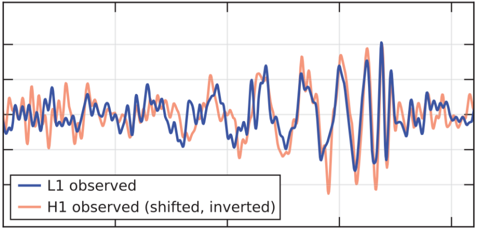
- So far, only weak-field precision tests
- Lots of theories $\approx$ GR
- Need to explore strong-field
  • Strong curvature • non-linear • dynamical

[Baker, Psaltis, Skordis (2015)]

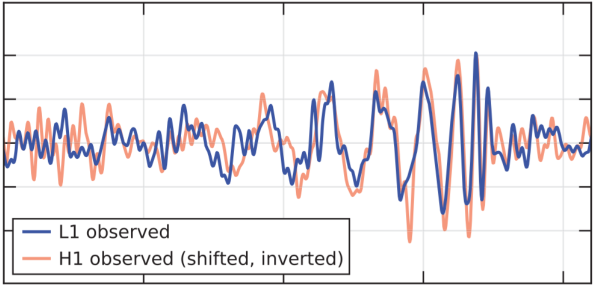


Vision

- Before this year: precision tests of GR in weak field
- Now: first direct measurements of dynamical, strong field regime



LIGO's tests



Selected for a Viewpoint in *Physics*

PRL **116,** 221101 (2016) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2016

Tests of General Relativity with GW150914

B. P. Abbott *et al.**

(LIGO Scientific and Virgo Collaborations)

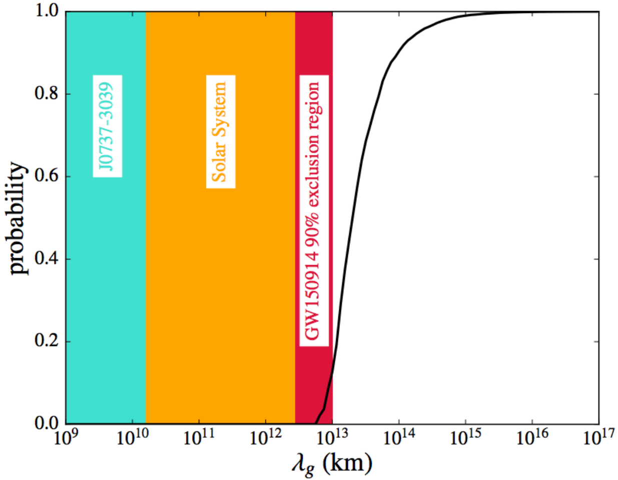
(Received 26 March 2016; revised manuscript received 9 May 2016; published 31 May 2016)

The LIGO detection of GW150914 provides an unprecedented opportunity to study the two-body

LIGO's tests

Two tests I like:

- Any deviation from GR must be below 4% of signal power
- Test of dispersion relation



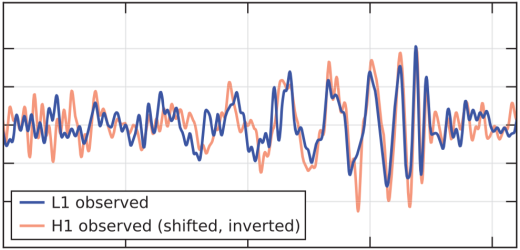
LIGO's tests

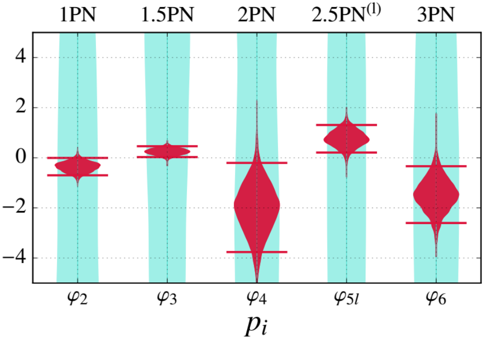
One test I do not particularly like:

- Insert power-law corrections to amplitude and phase ($u^3 \equiv \pi \mathcal{M} f$)

$$\tilde{h}(f) = \tilde{h}_{GR}(f) \times (1 + \alpha u^a) \times \exp[i\beta u^b]$$

- Parameters: (α, a, β, b)
- Inspired by post-Newtonian calculations in beyond-GR theories





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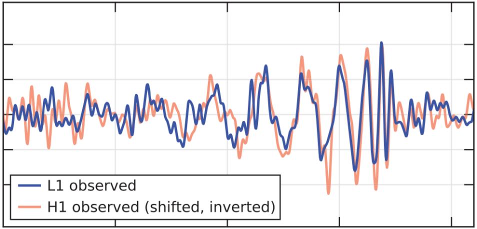

- Future: precision tests of GR in the strong field
  - Changing nuclear EOS is degenerate with changing gravity
  - Need black hole binary merger for precision

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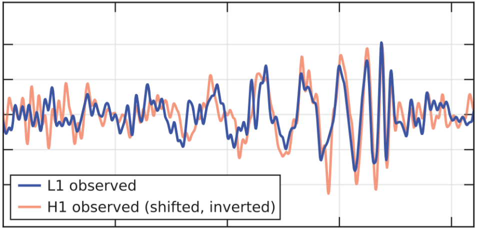

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Question: How to perform precision tests of GR in strong field?

How to perform precision tests

- Two approaches: theory-specific and theory-agnostic
- Agnostic: parameterize, e.g. PPN, PPE

Only 10 numbers in parametrized post-Newtonian [Slide from Wex]

PPN formalism for metric theories of gravity

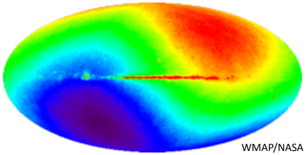
Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

w: motion w.r.t. preferred reference frame


WMAP/NASA

Metric potentials:

$$U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad \text{(Newtonian potential)}$$

$$U_{ij} = \int \frac{\rho'(x - x')_i (x - x')_j}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|}\right) d^3x'\, d^3x'',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

$$\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad \Phi_2 = \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad \Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad \Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$V_i = \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')](x - x')_i}{|\mathbf{x} - \mathbf{x}'|^3} d^3x'.$$

[Will 1993, *Will 2014, Living Reviews in Relativity*]

Parameterized post-Einstein framework

- Insert power-law corrections to amplitude and phase ($u^3 \equiv \pi \mathcal{M} f$)

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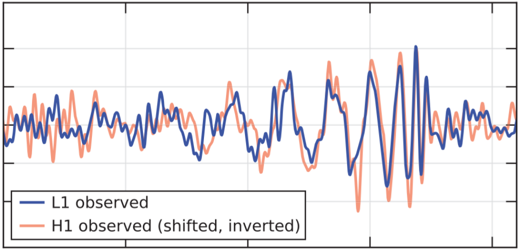

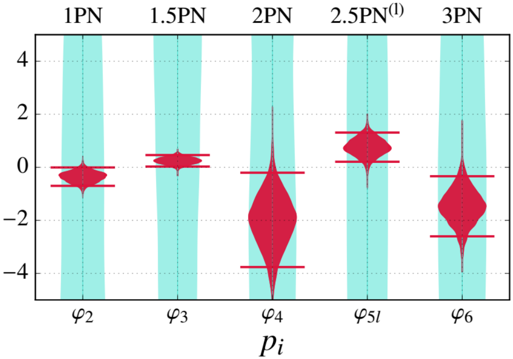

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Problem: Only simulated BBH mergers in GR!

The problem

From Lehner+Pretorius 2014:

> redshifts of $z \simeq 20$ with a SNR ≥ 10. For a recent review see Seoane et al. (2013).] Compounding the problem, despite the large number of proposed alternatives or modifications to general relativity (see, for example, Will 1993, 2006), almost none have yet been presented that (*a*) are consistent with general relativity in the regimes where it is well tested, (*b*) predict observable deviations in the dynamical strong field relevant to vacuum mergers, and (*c*) possess a classically well-posed initial value problem to be amenable to numerical solution in the strong field. The notable exceptions are a subset of scalar tensor theories, though these require a time-varying cosmological scalar field for binary black hole systems (Horbatsch & Burgess 2012) or one or more neutron stars in the merger (see Section 5). Thus there is little guidance on what reasonable strong-field deviations one might expect. Proposed solutions to (at least partially) circumvent these problems include the parameterized post-Einsteinian and related frameworks (Yunes & Pretorius

Don't know if other theories have good initial value problem

Numerical relativity

Numerical relativity

- Nonlinear, quasilinear, 2nd order hyperbolic PDE,
  10 functions, 3+1 coordinates
- Attempts from '60s until 2005.
  Numerically merging BHs for 11 years!
- Want to evolve. How do you know if good IBVP?
- Both under- and over-constrained.
    - gauge
    - constraints (not all data free; need constraint damping)

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Every other gravity theory will have at least these difficulties

Some other theories

"Scalar-tensor":

$$G^{\star}_{\mu\nu} = 2\left(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\star}_{\mu\nu}\partial_\sigma\varphi\partial^\sigma\varphi\right) - \frac{1}{2}g^{\star}_{\mu\nu}V(\varphi) + 8\pi T^{\star}_{\mu\nu}$$

$$\Box_{g^\star}\varphi = -4\pi\alpha(\varphi)T^\star + \frac{1}{4}\frac{dV}{d\varphi}$$

BBH in S-T:

- Massless scalar $\implies \varphi \to 0$, agrees with GR
- Only differ if funny boundary or initial conditions

Some other theories

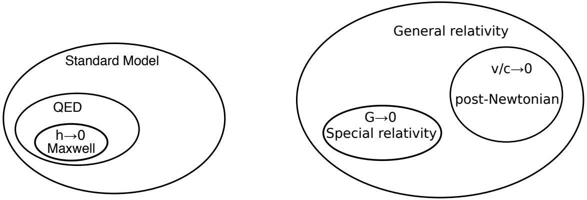
Higher derivative EOMS.

- Ostrogradski instability. H unbounded below
- Some theories try to avoid, e.g. Horndeski, dRGT
- If not quasi-linear, may violate Lipschitz continuity $\Longrightarrow$ Picard–Lindelöf theorem (ODE language); Cauchy–Kovalevskaya theorem is for quasi-linear
- Massive gravity theories. B-D ghost, cured by dRGT.

A solution

A solution

- Treat every theory as an effective field theory (EFT)
- Already do this for GR. Valid below some scale
- Theory only needs to be approximate, approximately well-posed



- Example: weak force below EWSB scale (lose unitarity above)

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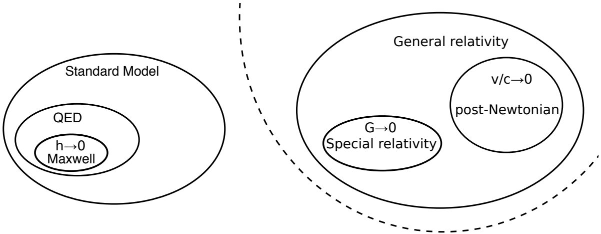

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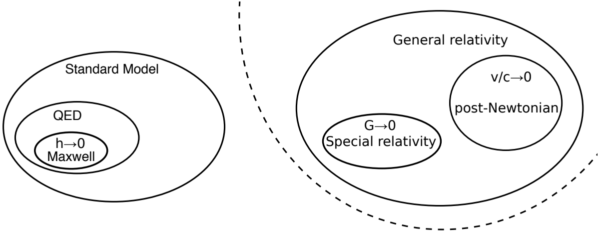

- Same should happen in gravity EFT:
  lose predictivity (bad initial value problem) above some scale
- Theory valid below cutoff $\Lambda \gg E$. Must recover GR for $\Lambda \to \infty$.
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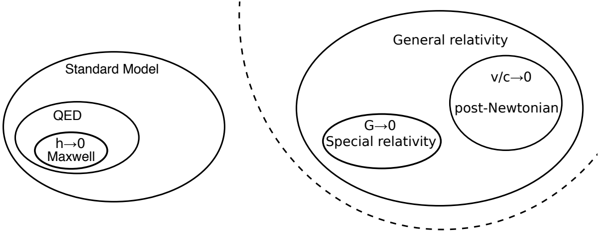

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Example: Dynamical Chern-Simons gravity

What is dynamical Chern-Simons gravity?

- Chern-Simons = GR + pseudo-scalar + interaction

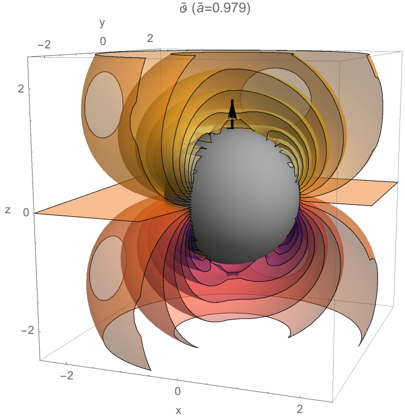
$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\vartheta)^2 + \varepsilon\,\vartheta\;{}^*RR\right]$$

$$\Box\vartheta = \varepsilon\,{}^*RR\,, \qquad\qquad G_{ab} + \varepsilon\,C_{ab}[\partial\vartheta\partial^3 g] = T_{ab}$$

- Anomaly cancellation, low-E string theory, LQG...
  (see Nico's review Phys. Rept. **480** (2009) 1-55)
- Lowest-order EFT with parity-odd ϑ, shift symmetry (long range)
- Phenomenology unique from other R^2 (e.g. Einstein-dilaton-Gauss-Bonnet)

Black holes in dCS

- $a = 0$ (Schwarzschild) is exact solution with $\vartheta = 0$
- Rotating BHs have dipole+ scalar hair



LCS, PRD **90** 044061 (2014) [arXiv:1407.2350]

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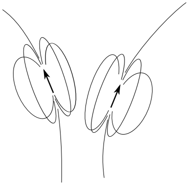
- Post-Newtonian of BBH inspiral in
  PRD **85** 064022 (2012) [arXiv:1110.5950]



- More updated phenomenology in
  CQG **32** 243001 (2015) [arXiv:1501.07274]

Back to problem and solution

- DCS had principal part $\partial^3 g$ coming from C_{ab} tensor. *Probably* not well-posed, Delsate+ PRD **91**, 024027.
- Theory is GR + $\varepsilon \times$ deformation. Expand everything in ε
- Derivation on board (if there's time, otherwise see me later)
- At every order in ε, principal part is $\text{Princ}[G_{ab}]$

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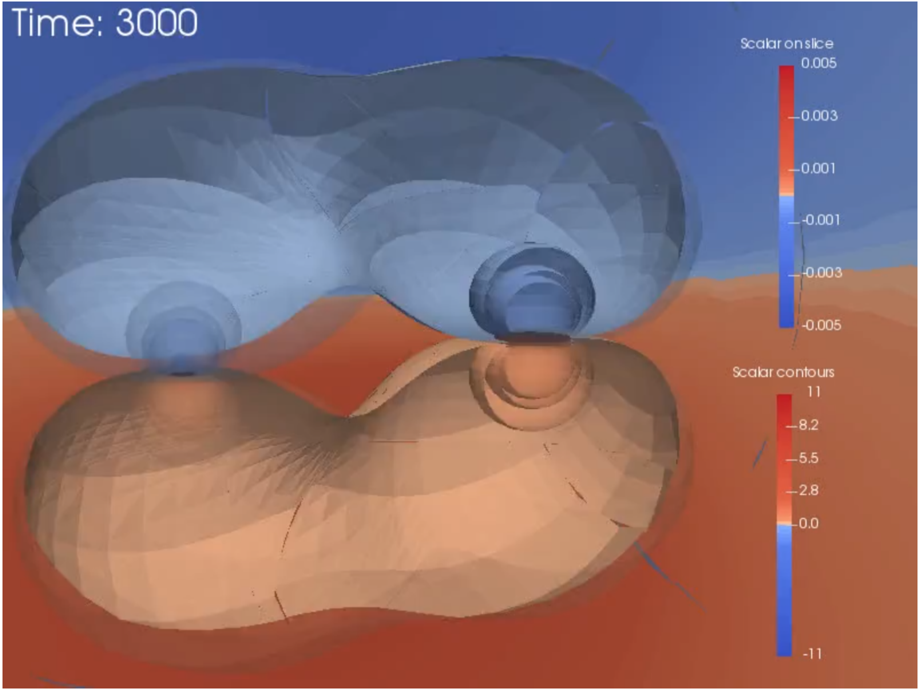
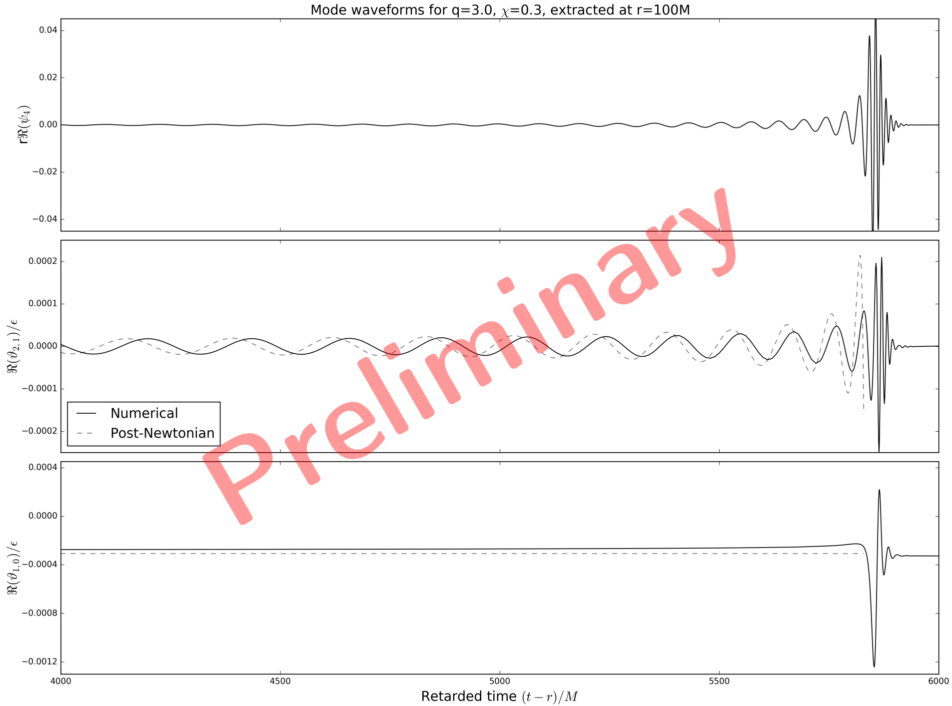
Background dynamics are well-posed $\Longrightarrow$ perturbations well-posed


Time: 3000
Scalar on slice
0.005
0.003
0.001
-0.001
-0.003
-0.005
Scalar contours
11
8.2
5.5
2.8
0.0
-11


Mode waveforms for q=3.0, χ=0.3, extracted at r=100M
0.04
0.02
0.00
−0.02
−0.04
$r\Re(\psi_4)$
0.0002
0.0001
0.0000
−0.0001
−0.0002
$\Re(\vartheta_{2,1})/\epsilon$
Numerical
Post-Newtonian
0.0004
0.0000
−0.0004
−0.0008
−0.0012
$\Re(\vartheta_{1,0})/\epsilon$
4000
4500
5000
5500
6000
Retarded time $(t-r)/M$
Preliminary

Next steps

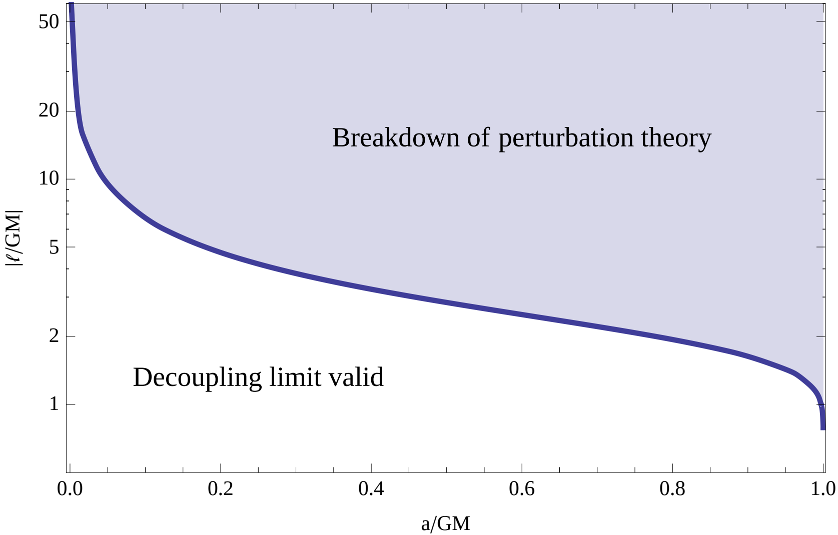
Go to $\mathcal{O}(\epsilon^2)$.

- *Two* sets of gauges, constraints
- Find stable gauge
  - Linearization of damped harmonic
- Regime of validity of perturbation scheme

$$\left\| \frac{\epsilon^2}{2} h^{(2)} \right\| \ll \left\| g^{(0)} \right\|$$

- Renormalization? See e.g. Galley and Rothstein [1609.08268]

Regime of validity



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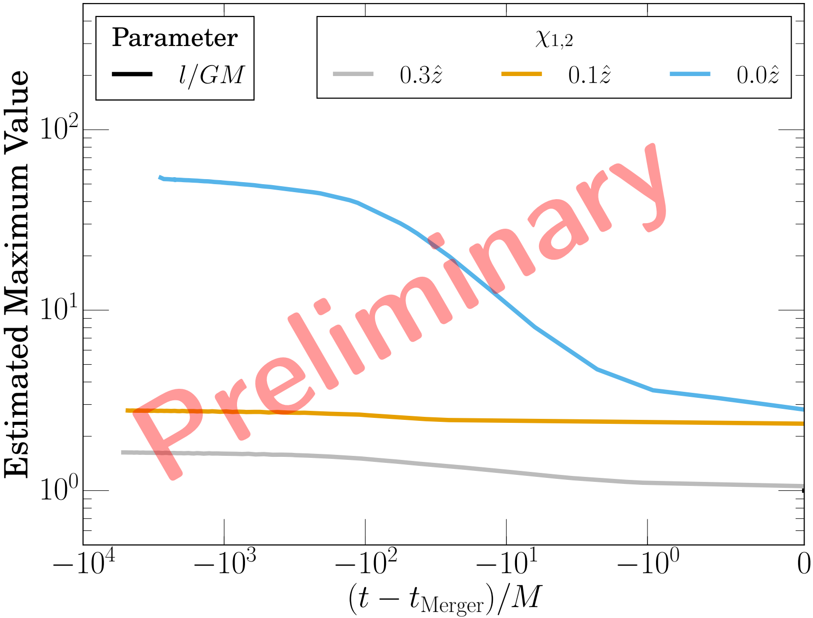

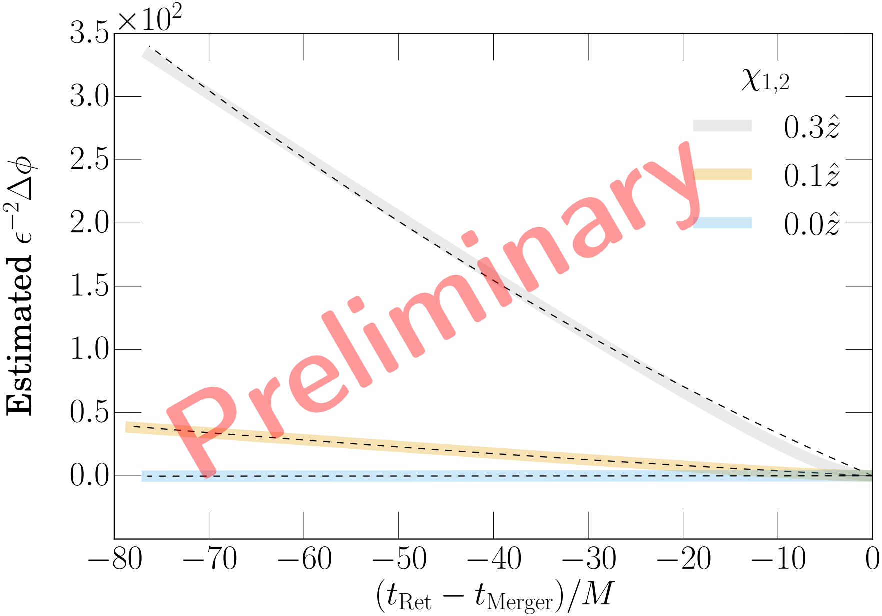

$\times 10^2$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Estimated $\epsilon^{-2}\Delta\phi$
$\chi_{1,2}$
$0.3\hat{z}$
$0.1\hat{z}$
$0.0\hat{z}$
Preliminary
−80
−70
−60
−50
−40
−30
−20
−10
0
$(t_{\mathrm{Ret}} - t_{\mathrm{Merger}})/M$

The TAKE AWAY™

- General relativity must be incomplete
- New opportunity to test GR in strong-field
- Present tests' shortcomings
  - Theory-independent tests need more guidance
  - Almost no theory-specific tests
- Challenge: Find spacetime solutions in theories beyond GR
  - Our contribution: First binary black hole mergers in dynamical Chern-Simons gravity
  - Method is generic, will work for many theories