UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Advanced Mechanics I (Phys. 709) — Prof. Leo C. Stein — Fall 2018

Problem Set 1

Due: Friday, Aug. 31, 2018, by 5PM

Here, FW = Fetter and Walecka, and HF = Hand and Finch. Reading FW §1.3 and §1.4 will be helpful for some of the problems. As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.

1–4. FW1.2, FW1.4, FW1.6, FW1.9

5. **Potentials with scaling properties** (Based on HF1.8). Consider a potential $V(\vec{r}_1, \ldots, \vec{r}_M)$ which has the property (for any positive α)

$$V(\alpha\vec{r}_1, \ldots, \alpha\vec{r}_M) = \alpha^k V(\vec{r}_1, \ldots, \vec{r}_M)\,. \tag{1}$$

This is called a homogeneous function, of degree k. Consider an action with this potential and with standard kinetic terms for the M particles of equal mass

$$S = \int \left[\left(\frac{m}{2} \sum_{i=1}^{M} \frac{d\vec{r}_i}{dt} \cdot \frac{d\vec{r}_i}{dt} \right) - V(\vec{r}_1, \ldots, \vec{r}_M) \right] dt\,. \tag{2}$$

(a) What is the transformed action S' if we simultaneously scale distances and times according to $\vec{r}_i{}' = \alpha\vec{r}_i, t' = \beta t$, for positive α, β? (Hint: pay attention to time derivative and the integration measure)

(b) How should β be related to α in order for the transformed action to yield the same equations of motion?

(c) What is the value of k for (i) a uniform gravitational potential, (ii) a simple harmonic oscillator, and (iii) the Kepler problem?

(d) What does this mean for oscillation frequencies in a simple SHO, and orbital frequencies in the Kepler problem?

6. **Functionals with more derivatives**. Suppose we have a functional that depends on k time derivatives of a function,

$$F[f(x)](x_1, x_2) = \int_{x_1}^{x_2} \mathcal{F}\left(f, \frac{\partial f}{\partial x}, \ldots, \frac{\partial^k f}{\partial x^k} \right) dx\,. \tag{3}$$

Suppose we vary $f(x) = f_0(x) + \epsilon\delta f(x)$ with endpoints fixed, $\delta f(x_1) = 0 = \delta f(x_2)$. Then what is the generalization of the Euler-Lagrange equation that results from this functional?