UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

Problem Set 5

Due: Monday, Oct. 7, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. **Surface charge on a non-uniform disk**. We've arranged charge to lie on a disk of radius R in the $z = 0$ plane. The surface charge density for a radius s away from the origin, where $0 \le s \le R$, is

$$\sigma(s) = \sigma_0 + \frac{s}{R}(\sigma_R - \sigma_0)\,, \tag{1}$$

   where σ_0 is the surface charge density at the center, and σ_R is the surface charge density at the edge. Find the potential $V(0, 0, z)$ along the z axis at some height z above the disk. (If you look up an integral in a table such as Gradshteyn and Ryzhik, include the reference to the identity you used; Mathematica or another computer algebra system will not count.)

2. **Cone of charge** (Griffiths 2.26 in 3rd ed.). We place a uniform surface charge density σ on the surface of a cone (like a hollow ice cream cone, with no top). The cone has a height h and the radius at the top is also h. Find the potential difference between point $\boldsymbol{a}$ (the vertex) and point $\boldsymbol{b}$ (the center of the top).

3. **Energy of a non-uniform spherical charge distribution**. Distribute charge onto a sphere of radius R so that the charge density is

$$\rho(r) = \begin{cases} \rho_R \frac{r^2}{R^2} & 0 \le r \le R \\ 0 & \text{otherwise}\,, \end{cases} \tag{2}$$

   where ρ_R is the volume charge density at the edge.

   (a) Compute the potential V and electric field $\boldsymbol{E}$ everywhere.
   (b) Compute the energy of this charge distribution, using V and ρ.
   (c) Compute the energy of this charge distribution, using $\boldsymbol{E}$.

4. **Two conductors**. We take one solid conducting ball of radius r_1, and place it inside a hollow conducting shell of radius r_2 (with negligible thickness). Now we place a charge $+q$ on the inner ball, and charge $-q$ on the outer shell, and let the charges come to static equilibrium.

   (a) Find the potential V and electric field $\boldsymbol{E}$ everywhere.
   (b) Compute the energy required to assemble this charge distribution, by using V and ρ
   (c) Compute the energy required to assemble this charge distribution, using $\boldsymbol{E}$.
   (d) Compute the capacitance of this geometry.