UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2020

Problem Set 2

Due: Weds., Feb. 12, 2020, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Griffiths problem 7.30 (Mutual inductance between two tiny wire loops).

2. Griffiths problem 7.53 (ratio of EMFs in a transformer).

3. Suppose we have two circuits – of any geometry! – at a distance R apart from each other. As R increases, the mutual inductance M will change as a function of R. Make an argument for the asymptotic behavior of M as a function of R at very large R, for example some function like $M(R) \sim e^{-R}$ (this is not the answer, but just demonstrating that we're looking for an asymptotic function).

4. Much of the interstellar medium (ISM) is very low number density, typically $n \approx 1\text{atom}/\text{cm}^3$ (you can assume it is entirely Hydrogen). There are magnetic fields permeating the galaxy with strengths like $B \approx 10\mu G$ (microGauss).
   (a) What is a typical energy density of the magnetic field in the ISM?
   (b) Suppose there is *equipartition* of energy between magnetic energy density and thermal energy density (which of course depends on density and temperature). What is a typical temperature of the ISM?

5. **A highly conducting, magnetized plasma**. Consider a plasma with a conductivity[1] σ, charge density ρ (that varies throughout the plasma), and where at each point the particles are moving with velocity $\boldsymbol{v}$ (that also varies from place to place). Ohm's law says that the current density is

$$\boldsymbol{J} = \sigma \boldsymbol{f} = \sigma \left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \right). \tag{1}$$

   (a) Suppose the conductivity σ is taken to infinity, while the current density $\boldsymbol{J} = \rho \boldsymbol{v}$ remains finite. What relationship does this imply between the electromagnetic fields?
   (b) From the previous answer, what do you know about $\boldsymbol{E} \cdot \boldsymbol{B}$?
   (c) Write $\boldsymbol{v}$ as the sum of two vectors, $\boldsymbol{v}_\parallel$ and $\boldsymbol{v}_\perp$ which are parallel and perpendicular to $\boldsymbol{B}$. Find an expression for $\boldsymbol{v}_\perp$ terms of $\boldsymbol{E}$ and $\boldsymbol{B}$.

   This scenario is actually applicable to many astrophysical plasmas! We will continue with this problem another week.

6. Griffiths 7.47 (3^{rd} edition) [7.49 in the 4^{th} edition] (getting $\boldsymbol{E}$ in terms of the vector potential).

[1] We will not refer to the resistivity, $1/\sigma$, which is sometimes denoted ρ. Instead we reserve the symbol ρ for the charge density.